2018-10-22-1

Last time: $\lambda = L(x)i$

$$V = \frac{d\lambda}{dt} = L(x)\frac{di}{dt} + \frac{\partial L}{\partial x}\frac{dx}{dt}i$$

Today: Force produced by electric field to move $x \Rightarrow f^e$

* Lots of Math!!

* Obtain f^e from force-energy relation.
* Recall for a conservative force: $\underline{F} = -\nabla\phi$ where ϕ is some potential

Ex) $\phi = mgy$

$\underline{F} = -mg\hat{y}$

We want to use the same idea here.

* Change in energy stored in field:

$$\Delta W_m = W_{in} - W_{used}$$

* Rate of change in energy stored:

$$\frac{dW_m}{dt} = \dot{W}_{in} - \dot{W}_{used}$$

Rate of energy in: $p = Vi = i\frac{d\lambda}{dt}$

Rate of energy used: $f^e\frac{dx}{dt}$

$$\boxed{\frac{dW_m}{dt} = i\frac{d\lambda}{dt} - f^e\frac{dx}{dt}}$$

* λ and x are independent variables

* $\frac{dW_m}{dt} = \frac{\partial W_m}{\partial \lambda}\frac{d\lambda}{dt} + \frac{\partial W_m}{\partial x}\frac{dx}{dt} \Rightarrow \boxed{i = \frac{\partial W_m}{\partial \lambda}, \quad f^e = -\frac{\partial W_m}{\partial x}}$

* Once W_m is known, f^e can be found!

2018-10-22-2

$$\frac{dW_m}{dt} = i\frac{d\lambda}{dt} - f^e\frac{dx}{dt}$$

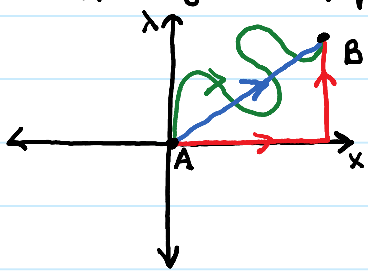
$$\Rightarrow dW_m = i\,d\lambda - f^e dx$$

Integrate and use dummy variables: $\displaystyle\int_{W_{m0}}^{W_m} dW_m^* = \int_{\lambda_0}^{\lambda} i\,d\lambda^* - \int_{x_0}^{x} f^e dx^*$

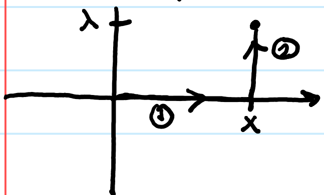
$$\Rightarrow W_m - W_{m0} = \int_{\lambda_0}^{\lambda} i\,d\lambda^* - \int_{x_0}^{x} f^e dx^*$$

* We assumed lossless fields $\Rightarrow$ conservative forces/fields

$\Rightarrow$ Energy change is independent of path taken!



* Pick path that is easiest for evaluating the integrals. (Here, red path)



* on ①, λ is constant $=0 \Rightarrow \displaystyle\int_0^0 i\,d\lambda^* = 0$, so only x contribution.

Assume if $\lambda = 0$, $f^e = 0$ so $\displaystyle\int_0^x f^e dx^* = 0$.

$\Rightarrow$ ① adds nothing to W_m

* on ②, x is constant $\Rightarrow \displaystyle\int_x^x f^e dx = 0$, so only λ contribution.

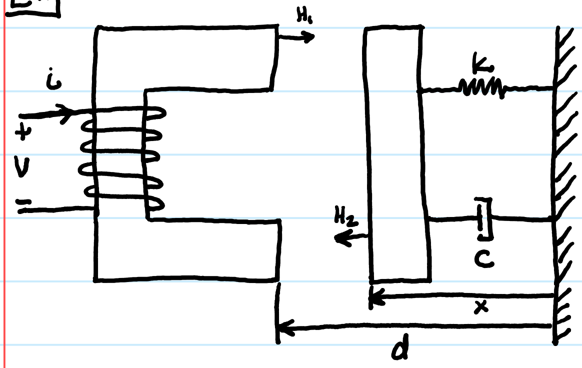
$W_m - W_{m0} = \displaystyle\int_0^{\lambda} i\,d\lambda^*$. Assume $W_{m0} = 0$ $\Rightarrow$ $\boxed{W_m = \displaystyle\int_0^{\lambda} i\,d\lambda}$

if $\lambda = L(x)\,i \Rightarrow i = \dfrac{\lambda}{L(x)}$ then $W_m = \displaystyle\int_0^{\lambda} \frac{\lambda^*}{L(x)}d\lambda^*$ $\Rightarrow$ $\boxed{\begin{array}{l} W_m = \frac{1}{2}\frac{\lambda^2}{L(x)} \\ f^e = \frac{1}{2}\left(\frac{\lambda}{L(x)}\right)^2\frac{\partial L}{\partial x} \\ \text{for } i = \frac{\lambda}{L(x)} \end{array}}$

$$f^e = -\frac{\partial W_m}{\partial x} = +\frac{1}{2}\frac{\lambda^2}{L(x)^2}\frac{\partial L}{\partial x}$$

2018-10-22-3

$\boxed{\text{Ex}}$



$$\boxed{\phi = \left[\frac{\mu_0 A N}{2(d-x)}\right] i}$$

$$\lambda = N\phi \Rightarrow \lambda = \left[\frac{\mu_0 A N^2}{2(d-x)}\right] i \qquad i = \frac{2(d-x)}{\mu_0 A N^2} \lambda$$

$$W_m = \int_0^{\lambda} i \, d\lambda^* \Rightarrow W_m = \int_0^{\lambda} \frac{2(d-x)}{\mu_0 A N^2} \lambda^* \Rightarrow W_m = \frac{1}{2}\left(\frac{2(d-x)}{\mu_0 A N^2}\right) \lambda^{*2} \Big|_0^{\lambda}$$

$$\boxed{W_m = \frac{1}{2}\left(\frac{2(d-x)}{\mu_0 A N^2}\right) \lambda^2}$$

$$f^e = -\frac{\partial W_m}{\partial x} = -\left(\frac{1}{2}\left[\frac{-2}{\mu_0 A N^2}\right] \lambda^2\right)$$

$$\boxed{f^e = \frac{1}{\mu_0 A N^2} \lambda^2 = \frac{1}{2} \frac{\mu_0 A N^2}{2(d-x)^2} i^2}$$

$$i = \frac{\partial W_m}{\partial \lambda} \Rightarrow i = \frac{2(d-x)}{\mu_0 A N^2} \lambda \quad \checkmark$$